

Vocabulary

- * boundary
- * dense
- * nowhere dense
- * bounded set
- * continuous function
- * uniformly continuous function

Homework

Examples:

DENSE SETS

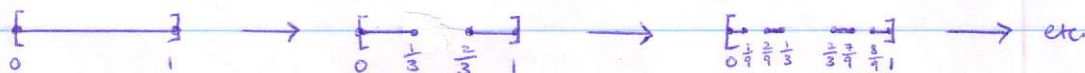
- (1) \mathbb{Q} is dense in \mathbb{R}
- (2) $[0, 1]$ is dense in $[0, 1]$

BOUNDARIES

- (1) The boundary of \mathbb{Q} in \mathbb{R} is \mathbb{R}
- (2) The boundary of $[0, 1]$ in \mathbb{R} is $\{0, 1\}$

NOWHERE DENSE SETS

- (1) $\mathbb{Z}_{>0}$ and \mathbb{Z} are nowhere dense in \mathbb{R}
- (2) \mathbb{R} is nowhere dense in \mathbb{R}^2
- (3) The Cantor set is nowhere dense in $[0, 1]$



BOUNDED SET

$\mathbb{Z}_{>0}$ is not bounded in \mathbb{R}

Homework

- Let (X, d) be a metric space and let $\vec{x}: \mathbb{Z}_{>0} \rightarrow X$ be a sequence in X . If \vec{x} converges then $\{x_1, x_2, \dots\}$ is bounded. $n \mapsto x_n$
- Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be continuous. Show that $g \circ f: X \rightarrow Z$ is continuous.

• Let $A \subseteq X$ and let $f: X \rightarrow Y$ be continuous. Show that $g: A \rightarrow Y$ is continuous. $a \mapsto f(a)$

Homework (continued)

Let $f_1: X_1 \rightarrow Y_1$ and $f_2: X_2 \rightarrow Y_2$ be continuous. Show that $f: X_1 \times X_2 \rightarrow Y_1 \times Y_2$ is continuous.
 $(x_1, x_2) \mapsto (f(x_1), f(x_2))$

Show that $\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is continuous.
 $(x, y) \mapsto x+y$

Show that $\mathbb{R} \rightarrow \mathbb{R}$ is continuous
 $x \mapsto -x$

Show that $\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is continuous.
 $(x, y) \mapsto xy$

Vocabulary

* dense
* nowhere dense
* bounded set

* continuous
* uniformly continuous

Examples

DERIVED SETS

(1) \mathbb{R} is dense in \mathbb{R}

(2) $[0, 1]$ is dense in $[0, 1]$

BOUNDARIES

(1) The boundary of \mathbb{Q} in \mathbb{R} is \mathbb{R}

(2) The boundary of $[0, 1]$ in \mathbb{R} is $\{0, 1\}$

NON-EMPTY DENSE SETS

(1) \mathbb{Z} and \mathbb{Q} are nowhere dense in \mathbb{R}

(2) \mathbb{R} is nowhere dense in \mathbb{R}

(3) The Cantor set is nowhere dense in $[0, 1]$



BOUNDED SETS

\mathbb{Z} is not bounded in \mathbb{R}

Homework

Let (X, d) be a metric space and let $\{x_n\} \subset X$ be a sequence in X . If $\{x_n\}$ converges then $\{x_n\}$ is bounded.

Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be continuous. Show that $g \circ f: X \rightarrow Z$ is continuous.

Let $A \subseteq X$ and let $f: X \rightarrow Y$ be continuous. Show that $f|_A: A \rightarrow Y$ is continuous.