Lecture 10 Summary

Wed 13/08/14

Vocabulary

* boundary
* dense
* nowhere dense
* bounded set
* continuous function
* uniformly continuous function

Examples:

DENSE SETS

(1) $\mathbb{Q}$ is dense in $\mathbb{R}$

(2) $(0,1]$ is dense in $[0,1]$

BOUNDARIES

(1) The boundary of $\mathbb{Q}$ in $\mathbb{R}$ is $\mathbb{R}$

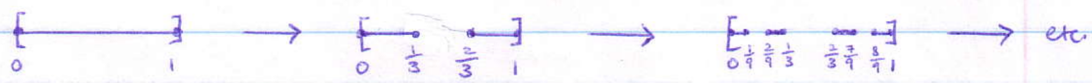
(2) The boundary of $(0,1]$ in $\mathbb{R}$ is $\{0,1\}$

NOWHERE DENSE SETS

(1) $\mathbb{Z}_{>0}$ and $\mathbb{Z}$ are nowhere dense in $\mathbb{R}$

(2) $\mathbb{R}$ is nowhere dense in $\mathbb{R}^2$

(3) The cantor set is nowhere dense in $[0,1]$



BOUNDED SET

$\mathbb{Z}_{>0}$ is not bounded in $\mathbb{R}$

Homework

* Let (X,d) be a metric space and let $\vec{x}: \mathbb{Z}_{>0} \to X$, $n \mapsto x_n$ be a sequence in X. If $\vec{x}$ converges then $\{x_1, x_2, \dots\}$ is bounded.

* Let $f: X \to Y$ and $g: Y \to Z$ be continuous. Show that $g \circ f: X \to Z$ is continuous.

* Let $A \subseteq X$ and let $f: X \to Y$ be continuous. Show that $g: A \to Y$, $a \mapsto f(a)$ is continuous.

Homework (continued)

- Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ be continuous. Show that $f : X_1 \times X_2 \to Y_1 \times Y_2$ is continuous.
  $(x_1, x_2) \mapsto (f(x_1), f(x_2))$

- Show that $\mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is continuous.
  $(x, y) \mapsto x + y$

- Show that $\mathbb{R} \to \mathbb{R}$ is continuous
  $x \mapsto -x$

- Show that $\mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is continuous.
  $(x, y) \mapsto xy$

7mm A4